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Modeling of green body strength, internal pressure, and stress in porous ceramic bodies during thermal debinding

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A model is developed to describe the evolution of pressure, stress, and green body strength in porous ceramic bodies during the thermal debinding heating cycle. Pressure in the green body arises from the thermally activated decomposition of the binder, and the pressure increase is mitigated by flow through the porous medium. The stress then arises from the pressure gradient within the green ceramic body. The strength model incorporates the effects of solids and binder loading, and the effect of temperature on the yield behavior of the binder. The stress is then compared to the strength of the green body during the heating cycle in order to determine when failure occurs. The assumptions, the necessary experimental data, and the weaknesses in the model are discussed.

Key Words: thermal debinding, binder removal, failure, binders

Introduction

During the processing of ceramic green bodies, additions of binder not only aid in the handling and subsequent processing of the green component, but the binder may also ultimately control the failure behavior during thermal debinding [1, 2]. In general, the failure behavior of ceramic green bodies can be succinctly summarized by stating that failure will occur when the local stress [3-5] within the green body exceeds the local strength. The failure behavior of the green body in turns affects the duration of the debinding heating cycle. For example, when the strength of the green body is sufficiently large to withstand the stresses that accompany pressure buildup during thermal debinding, the heating cycle can be short. Conversely, if the green body is weak, the heating cycle must be correspondingly long so as to minimize pressure buildup and stress and thereby avoid failure of the green component.

In order to be able to compare the strength of a green body with the stress during binder removal, three problems must be addressed, namely: 1) the spatial distribution of pressure within the green body; 2) the spatial distribution of stress arising from the pressure field; and 3) the strength of the green body. Each of these problems in and of itself is fairly difficult to model, the reasons for which are enumerated below, but, in addition, the difficulty of each is exacerbated because the properties of the green body are changing with time and temperature during the thermal debinding heating cycle.

The first problem-the spatial distribution of pressure

[3-11]— is a coupled function of the temperature distribution in the green body, the decomposition kinetics of the binder, and the permeability of the green body, all of which are changing during the heating cycle. For the case of openpore green compacts, modeling of these coupled kinetic and transport processes indicates that the pressure is largest at the center of the green body and decreases towards the body edges [4, 5].

The second problem—the spatial distribution of stress [3-5] arising from the pressure—has also been examined, but in far fewer studies. To obtain the stress from the pressure, it is necessary to prescribe values of the material properties of the green body, and this has been generally accomplished by incorporating linear elastic behavior into mechanics models. The outcome of these studies is that the stress state within the green body consists of both normal and shear stresses, and the former are larger by approximately an order of magnitude [5]. For open-pore compacts, the largest normal stresses are found in the center of the green body, which coincides with the location of the highest pressure.

Although a common assumption, constant linear elastic behavior of the green body throughout thermal debinding is unlikely to be realistic, because both the binder loading and the mechanical properties of the green body, via the properties of the binder, are changing with time and temperature. This unrealistic assumption likely has arisen because of the absence of reported stress-strain behavior of the green body as a function of binder loading and temperature.

Two key observations from experiment, however, may partially mitigate the use of linear elastic behavior for describing the mechanical properties of the green body. First, for green bodies with high organic loadings, the stress-strain behavior is likely to be proportional, either linearly or non-linearly, up to yielding [12-18]. Beyond

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to provide some qualitative and quantitative insight. The second observation from experiment is that the failure temperatures of ceramic green bodies often span a relatively narrow ~30 degree C temperature range early in the heating cycle, when the binder loading is still high and nearly constant [19-21]. Thus, a description of the mechanical properties of the green body over a wide range of conditions may not necessarily be required. In light of these two observations, linear elastic behavior may again suffice, and we can thus use the results from earlier work relating pressure [11] and stress [5] in ceramic green bodies.

important, and the use of linear elastic behavior may suffice

Finally, even if the pressure and accompanying stress state of the green body were known, little or no data have been reported on the strength of composite ceramic green bodies over a wide range of binder loading and temperature. In addition, little or no data have been presented on the elevated temperature mechanical properties of the commonly used organic constituents of the binder, either individually or in combination. This lack of data may have occurred because the binder is a fugitive species, and thus a full understanding of the mechanical properties has not been considered to be warranted. A second reason may be that because binder degradation is a kinetic process that occurs at elevated temperature, it is actually difficult to perform measurements that can be unambiguously related to the physical and mechanical state of the green body. Therefore, to describe the strength of green bodies during thermal debinding, we make recourse to simple models that capture the essential features of the mechanical behavior of composite bodies involving polymeric species, and thus use a model that incorporates the effects of time, temperature, and strain rate to predict the yield strength [12-18].

In spite of the difficulties enumerated above, it is the aim of this work to develop relationships between pressure and stress, and then to compare the stress to the strength of the composite green body as a function of temperature. We first develop a model that can be used to predict the strength of the green body throughout the thermal debinding cycle. We next relate the green strength during the heating cycle to the stress, which is obtained from the pressure distribution in the green body. The combination of the results from the strength, pressure, and stress models then allows us to make some qualitative assessments of how these important quantities are changing relative to each other during thermal debinding, and how they influence the observed failure behavior of green bodies during thermal debinding [19-26].

Model

Strength of the green body

To determine the strength of the green body, we start with



Fig. 1. Distribution of binder between ceramic particles for pendularstate bonds and coated-state bonds.

the combined model of Rumpf [27] and Onoda [28]. The former starts with the volume fractions of solids and voids in the green body, and then ascribes the strength of the green body to forces arising at localized point contacts. Onoda [28] then modified this view by adding to the green body finite amounts of binder distributed (see Fig. 1) either in the pendular state or as a spherical shell coating each ceramic particle. Although the specifics of the strength of the green body depended on the amount of binder and how the binder was distributed, both the pendular- and coated-state models behaved in a similar qualitative fashion, namely, in that the green strength decreased with both decreasing fractions of ceramic and binder within the green body. For binder distributed in the pendular state, the tensile strength of a green body, σ , can be expressed as [27, 28]:

$$\sigma = \frac{3\sqrt{3\pi}}{8} \frac{(\varepsilon_c + \varepsilon_b)}{\sqrt{(1 - \varepsilon_c - \varepsilon_b)}} \left(\frac{\varepsilon_b}{\varepsilon_c}\right)^{1/2} \sigma_b \tag{1}$$

where ε_b is the volume fraction of binder, ε is the volume fraction of pores, ε_c is the volume fraction of ceramic and σ_b , which is the cohesive (or adhesive strength of the binder), is taken as a constant, which can be assigned to the room temperature properties of the binder.

We now modify Eq. 1 because at elevated temperature, yielding of the polymeric species may occur, and it thus becomes necessary to incorporate the effects of both temperature, T, and strain rate, $\dot{\varepsilon}_y$, on the mechanical behavior of the binder. Several models are available [12-18], and for simplicity we adopt the model of Ree and Eyring [12], which is based on transition state theory. This model relates the yield strength of the polymer $\sigma_{b,y}$ to the strain rate and the activation energy, ΔH , as:

$$\sigma_{b,y} = T\left(\frac{2}{V^*}\right) \left[\frac{\Delta H}{T} + 2.303 R \log\left(\frac{\dot{\varepsilon}_y}{\dot{\varepsilon}_o}\right)\right]$$
(2)

The quantity V^* is the activation volume, ε_o is a constant, and *R* is the gas constant. Equation 2 can thus be used to represent the case in which yielding of polymeric or organic species occurs at elevated temperature. When Eq. 2 is used to represent the strength of the binder and is combined with Eq. 1, the strength of the green body is given by:

$$\sigma = \frac{3\sqrt{3\pi}}{8} \frac{(\varepsilon_c + \varepsilon_b)}{\sqrt{(1 - \varepsilon_c - \varepsilon_b)}} \left(\frac{\varepsilon_b}{\varepsilon_c}\right)^{1/2} T\left(\frac{2}{V^*}\right) \left[\frac{\Delta H}{T} + 2.303 R \log\left(\frac{\dot{\varepsilon}_y}{\dot{\varepsilon}_o}\right)\right]$$
(3)

Equation 3, which now contains the effects of both binder loading and temperature, can then be used to predict the strength of the green body throughout the thermal debinding cycle.

Internal pressure during binder removal

In earlier work, we have derived in detail the coupled kinetic and transport equations for describing the internal pressure in an open-pore green ceramic body during thermal debinding. We thus here only present the relevant equations and provide references to the earlier work.

The rate of binder degradation, r, can be represented as a first order process as:

$$r = -\frac{d\varepsilon_b}{dt} = A \exp\left[-\frac{E}{RT}\right]\varepsilon_b \tag{4}$$

where t is the time, A is the pre-exponential factor, and E is the activation energy. For a linear heating rate, β , the solution to Eq. 4 is [29]:

$$\frac{\varepsilon_{b}}{\varepsilon_{bo}} = \exp\left(-\frac{A}{\beta}\left\{\frac{\left(\frac{RT^{2}}{E}\right)\exp\left(-\frac{E}{RT}\right)}{1+\frac{2RT}{E}} - \frac{\left(\frac{RT_{o}^{2}}{E}\right)\exp\left(-\frac{E}{RT_{o}}\right)}{1+\frac{2RT_{o}}{E}}\right\}\right)$$
(5)

where ε_{bo} is the initial volume fraction of binder and T_o is the initial temperature.

During the heating cycle for thermal debinding, the normalized pressure in the center of the green body, $(P/P_o)_o$, can be expressed as [11]:

$$\left(\frac{P}{P_o}\right)_o \approx \left(1 + 0.8365 \frac{\mu}{2\rho_o^2 \kappa R} \frac{r\rho_b}{M} \frac{T}{T_o^2 L_x^2 L_y^2 + L_x^2 L_z^2} + L_y^2 L_z^2\right)^{1/2}$$
(6a)

$$\approx \left(1 + G\frac{rT}{\kappa}\right)^{1/2} \tag{6b}$$

where L_x , L_y , and L_z are the dimensions of the component, μ is the gas viscosity, ρ_b is the density of the binder, M is the binder molecular weight, and ρ_o is the initial gas density in the furnace. The permeability, κ , is given by the Kozey-Carman equation as:

$$\kappa = \frac{\varepsilon^3}{k(1-\varepsilon)^2 S^2} \tag{7}$$

where S is the specific surface, and k is a constant. Conservation of volume then relates the volume fractions of binder, porosity (ε), and ceramic as:

$$\varepsilon_b + \varepsilon + \varepsilon_c = 1 \tag{8}$$

Internal stress during binder removal

To describe the stress within the green body [5], we use a model, derived earlier [11], for describing the internal pressure in an open-pore green ceramic body during thermal debinding. The following assumptions are further made in the development of the stress model, namely, that the porous ceramic during thermal debinding consists of a void fraction, ε , and a solid skeleton fraction, $1-\varepsilon$. Secondly, the gas phase cannot support shear stresses, and both viscous and inertial stresses accompanying fluid flow are neglected. Finally, the solid skeleton is modeled as an isotropic linearelastic solid.

Force and moment equilibrium then leads to:

$$\frac{\partial \tilde{\sigma}_{xx}}{\partial x} + \frac{\partial \tilde{\sigma}_{xy}}{\partial y} + \frac{\partial \tilde{\sigma}_{xz}}{\partial z} = \varepsilon \frac{\partial P/P_o}{\partial x}$$
(9)

$$\frac{\partial \tilde{\sigma}_{xy}}{\partial x} + \frac{\partial \tilde{\sigma}_{yy}}{\partial y} + \frac{\partial \tilde{\sigma}_{yz}}{\partial z} = \varepsilon \frac{\partial P/P_o}{\partial y}$$
(10)

$$\frac{\partial \tilde{\sigma}_{xz}}{\partial x} + \frac{\partial \tilde{\sigma}_{yz}}{\partial y} + \frac{\partial \tilde{\sigma}_{zz}}{\partial z} = \varepsilon \frac{\partial P/P_o}{\partial z}$$
(11)

where $\tilde{\sigma}_{ii}$ and $\tilde{\sigma}_{ij}$, (*i* and *j* correspond to *x*, *y*, and *z*) are the components of nominal normal and nominal shear stress, respectively, on the skeleton. Equations 9-11 show that an increase in internal pressure causes stress, an effect that is proportional to the void fraction. The pressure gradients on the right-hand sides are thus equivalent to a body force throughout the porous green body. The nominal stresses, $\tilde{\sigma}$, are smaller than the true stresses, σ' , as given by $\tilde{\sigma}_{ij} = (1 - \varepsilon) \sigma'_{ij}$. To complete the description of the problem, the skeleton is assumed to follow isotropic elastic constitutive laws with Poisson's ratio, $\nu = 0.3$, and Young's modulus, $E = 2 \times 10^7$ N/m² [4]. Equations 9-11 were solved with a commercial finite element analysis program.

Results and Discussion

We first present results from Eq. 1, in which the strength of a green body depends on the volume fractions of both binder and ceramic and on the strength of the binder, which is assumed to have a constant tensile yield strength of $\sigma_b = 2.1$ MPa [30] at 300 K. Fig. 2 shows that for a given solids loading, the green strength increases strongly with binder loading. In addition, for a given binder loading, the green strength increases as the solids loading increases and this becomes more pronounced at high binder loadings as the volume fraction of solids exceeds 0.5.

The dependence of binder strength on temperature and strain rate as given by Eq. 2 is shown in Fig. 3. The yield strength decreases linearly with increasing temperature, the slope of which depends on the strain rate. For a constant temperature, decreasing the strain rate leads to a weaker green body. In addition to the dependence on the temperature and strain rate, both the activation energy and activation volume can influence the yield strength, as seen in Fig. 4.



Fig. 2. Strength (Eq. 1) versus binder volume fraction at different volume fractions of solids for pendular state bonds for $\sigma_b = 2.1$ MPa.



Fig. 3. Yield strength (Eq. 2) of binder versus temperature at different strain rates with $\Delta H = 30.4 \text{ kJ/mol}$, $\dot{\varepsilon}_o = 1 \times 10^6 \text{ s}^{-1}$, and $V^* = 14 \text{ m}^3/\text{kmol}$.

The combined effects (Eq. 3) of binder fraction and temperature on the green strength are presented in Fig. 5 for a fixed ceramic loading. The green body strength increases with binder fraction and decreases with increasing temperature. Fig. 5 also shows qualitatively a trajectory of the strength of a green body during a heating cycle from room temperature to some maximum temperature, T_{max} , followed by cooling to room temperature. For such a heating cycle, the green strength first decreases due to the degradation of binder and to the increase in temperature. Depending on where T_{max} is located in the heating cycle relative to the binder volume fraction, different behavior may arise. If T_{max} is achieved while the green body still retains binder, the strength may actually increase during the cooling branch, as is indicated by the dashed line in Fig. 5. For the more conventional case of the binder being completely degraded on the heating branch of the cycle, then the strength of



Fig. 4. Yield strength (Eq. 2) versus temperature at a constant strain rate of $\dot{\varepsilon}_y = 1800 \text{ s}^{-1}$ for different activation energies and activation volumes with $\dot{\varepsilon}_o = 1 \times 10^6 \text{ s}^{-1}$.



Fig. 5. Strength (Eq. 3) versus binder volume fraction at different temperatures with $\varepsilon_c = 0.5$, $\dot{\varepsilon}_y = 1800 \text{ s}^{-1}$, $\Delta H = 30.4 \text{ kJ/mol}$, $\dot{\varepsilon}_o = 1 \times 10^6 \text{ s}^{-1}$, and $V^* = 14 \text{ m}^3/\text{kmol}$. Also shown in the figure (dashed line with filled circles) is the trajectory of strength versus temperature for a heating cycle with a maximum temperature of 277 °C.

the green body decreases monotonically with decreasing volume fraction of binder. The specific strength trajectories realized during an actual debinding cycle will depend on the details of the heating cycle, which will be addressed next.

To probe the failure behavior of green bodies, a simple debinding procedure can be used whereby green bodies of different sizes are fabricated and then are subjected to a rapid linear heating rate. We first show in Fig. 6 how the pressure in the center of the green body evolves as a function of temperature during a linear heating rate for the model parameters in Table 1. The pressure first increases due to the enhanced rate of binder decomposition, then goes through a maximum, and finally decreases as the binder is degraded and more void space is created in the green body. The body of larger dimensions experiences a larger internal pressure,



Fig. 6. Normalized pressure in the center of the body and binder volume fraction versus temperature for a linear heating rate of $10 \text{ K} \cdot \text{minute}^{-1}$ for green bodies of two different sizes.

Table 1. Parameters used in the simulations

Symbol (units)	Value
P _o (MPa)	0.1
$T_o(K)$	300
M (kg/mol)	0.044
R (m ³ Pa/mol K)	8.314
μ (Pa s)	$2.5 imes 10^{-5}$
$S(m^{-1})$	$6 imes 10^5$
k (-)	430
$\rho_{\rm b} ({\rm kg/m^3})$	1000
ε _c (-)	0.55
ε _{bo} (-)	0.3
ε ₀ (-)	0.15
$\rho_{\rm o} ({\rm mol}/{\rm m}^3)$	40.09
A (s^{-1})	$1.7 imes10^4$
E (J/mol)	68000

as expected from the form of Eq. 6. Fig. 6 also shows the evolution of the volume fraction of binder during this heating cycle, where the maximum in pressure corresponds to decomposition of approximately 17% of the binder. For the two different sized bodies, the curves of ε_b are coincident, which can be seen from the form of Eq. 5.

We can now determine from the models (Eqs. 1-3) how the strength of the green body changes during such a linear heating ramp. Curve A in Fig. 7 shows the evolution of the green strength as given by Eq. 3, which incorporates the effects of changes in both binder loading and temperature. During the initial stage of the heating ramp, the strength of the green body decreases nearly linearly, which reflects the effect of temperature on the strength of the binder at a constant binder loading, which arises because the decomposition rate is too low. At the point in the heating cycle corresponding to the maximum in pressure, the green strength is ~25% of its initial value. Beyond the maximum in pressure, the green strength continues to decrease but now less slowly.



Fig. 7. Strength (Eq. 3 with $\Delta H = 30.4$ kJ/mol, $\dot{\varepsilon}_o = 1 \times 10^6$ s⁻¹, $V^* = 14$ m³/kmol, $\dot{\varepsilon}_v = 1800$ s⁻¹) versus temperature for a linear heating rate for three cases: A) Both ε_b and T vary during the heating cycle; B) Varying T and $\varepsilon_b = \varepsilon_{bo}$ during the heating cycle; and C) Varying ε_b and $T = T_o$ during the heating cycle.

To see more clearly the separate effects of binder decomposition and temperature on the evolution of green strength, two additional curves are shown in Fig. 7. The effect of temperature alone, while holding the binder loading constant at its initial value (see Curve B), leads to a slightly more gradual decrease in strength above 175 °C as compared to Curve A. If the strength of the binder is taken as constant at its room temperature value (Curve C), then the green body strength only depends on the volume fraction of binder, and it remains constant until appreciable binder begins to decompose. With further increases in temperature, the green strength ultimately drops to near zero in a sigmoidal fashion, which reflects the sigmoidal decrease in the volume fraction of binder with temperature (See Fig. 6).

Figs. 6 and 7 can thus be used to qualitatively compare the evolution of pressure to the strength of the green body. To make a quantitative determination of failure behavior, however, the strength of the green body must be compared to the stress, which we accomplish in the following fashion. The results from Ref. [5] relate the spatial distribution of stress in ceramic green bodies as a function of the spatial distribution of pressure arising from binder decomposition. The most important outcomes of this study were that both the maximum pressure and maximum normal stress occur in the center of the green body and that the normal stresses in each direction are approximately equal and are an order of magnitude larger than the shear stresses. We thus can represent, for a cube shaped green body of 2 cm side length, the maximum normal stress, σ_{xx} , in the body center as:

$$\log \sigma_{xx} \approx -0.0587 \left(\log \left[\frac{Gr}{\alpha \kappa T} \right] \right)^2 + 0.8876 \log \left[\frac{Gr}{\alpha \kappa T} \right] + 4.604$$
(12)

where:

$$\alpha = \left(\frac{0.8365}{T_{\rho}^2}\right) \frac{L_{\gamma}^2 L_{z}^2}{L_{\chi}^2 L_{\gamma}^2 + L_{\gamma}^2 L_{z}^2 + L_{\chi}^2 L_{z}^2}$$
(13)

Similar relationships are valid for green bodies of other sizes and aspect ratios.

Fig. 8 demonstrates the evolution of normal stress in the green body versus temperature for a sample subjected to a linear heating rate. The stress profiles are now seen to mimic the pressure profiles from Fig. 6, which is a direct consequence of the assumptions underlying Eq. 12. Even if these assumptions are overly restrictive, however, it is still likely that during thermal debinding, the stress initially rises, reaches a maximum at some point in the heating cycle, and then ultimately decreases as the pressure in the green body decreases.

We can now superimpose the results for the evolution of the strength of the green body versus the evolution of normal stress. Fig. 8 shows such behavior for three values of ΔH , which correspond to three room-temperature values of σ_b . In the first case, for a strong green body at room temperature, the stress in the green body never exceeds the strength, and thus the green body would survive the debinding cycle, regardless of size. For a green body of intermediate strength, the larger green body will fail near the maximum in pressure whereas the smaller green body will survive. Finally, for the weakest green body, both green bodies will fail relatively early in the heating cycle at high binder loading. This latter behavior is what was observed in Ref. [21], where green bodies of three sizes all failed early in the heating cycle over a 25 degree C temperature range.

A second type of procedure can be used to probe the failure behavior of green ceramic bodies during thermal debinding. In this case, bodies of constant size are subjected



Fig. 8. Stress in the center of the body (solid lines) and green body strength (dashed lines) versus temperature for a linear heating rate of 10 K·minute⁻¹ for green bodies of two different sizes of $L_i = 0.02$ m and Lx = Ly = 0.02 m, Lz = 0.002 m. Three cases for the strength are shown for $\Delta H = 25$, 30.4, and 35 kJ/mol with $\dot{\varepsilon}_o = 1 \times 10^6 \text{ s}^{-1}$, $V^* = 14 \text{ m}^3/\text{kmol}$, and $\dot{\varepsilon}_v = 1800 \text{ s}^{-1}$.

to heating cycles at different linear heating rates [21]. Fig. 9 shows the profiles of pressure and volume fractions of binder during the heating cycle for this case. The pressure profiles are now shifted to higher temperatures with increasing heating rate and larger maxima in pressure occur with higher heating rates. The volume fractions of binder are also shifted to higher temperature with increasing heating rate, and they are no longer coincident, as was seen in Fig. 6.

Fig. 10 shows the stress profiles for the same two heating rates as in Fig. 9. The stress profiles are also shifted to higher temperatures with larger maxima in stress occurring at higher heating rates. Comparisons of the strength of the green body and stress in the body center are also given in Fig. 10 for three values of ΔH , which corresponds to three room-temperature values of σ_b . Once again, the inter-



Fig. 9. Normalized pressure in the center of the body and binder volume fraction versus temperature for a green body of fixed size of $L_i = 0.02$ m at two linear heating rates.



Fig. 10. Stress in the center of the body (solid lines) and green body strength (dashed lines) versus temperature for a green body of fixed size of $L_i = 0.02$ m at two linear heating rates of 1 and 10 K minute⁻¹. Three cases for the strength are shown for $\Delta H = 25$, 30.4, and 35 kJ/mol with $\dot{\varepsilon}_{\alpha} = 1 \times 10^6 \text{ s}^{-1}$, $V^* = 14 \text{ m}^3/\text{kmol}$, and $\dot{\varepsilon}_y = 1800 \text{ s}^{-1}$.

section of the curves—which corresponds to failure—depends on the value of the strength of the binder. For the strongest green body, the component survives, regardless of the heating rate. For the intermediate strength case, the green body will fail near the maximum in pressure for the highest heating rate, but survive for the lower heating rate. For the weakest green body, the components will fail early in the heating cycle, regardless of the heating rate. This latter case has been observed in Ref. [21].

In summary, we have noted an absence of mechanical property data for green ceramic bodies during thermal debinding. To circumvent this shortage, we have combined different models in order to predict the strength of the green body as a function of temperature and binder loading. We have then compared the strength of the green body to the stress. This latter quantity we obtained by first incorporating the pressure distribution into a mechanics model, where we assumed linear elastic behavior. As mentioned earlier, this assumption may not be overly restrictive because up to yielding, stress and strain are likely to be proportional, and in addition, green bodies tend to fail over a narrow temperature range. In the stress model, we also did not account for the effect of temperature on the elastic modulus, which likely will impact the magnitude of the stress but may not influence the qualitative behavior of the stress reaching a maximum at some point in the heating cycle. The strength and stress profiles during thermal debinding were then compared in order to determine their intersection, which is when failure of the green body would occur. Several of the trends for green body failure seen in Figs. 8 and 10 are consistent with what has been observed in experiments.

Conclusions

In this work, we have examined during thermal debinding how the distribution of pressure in green bodies can lead to stress, and then compared the stress to the green strength of the composite ceramic body. To make this comparison requires use of three different models for which not all of the parameters are known with high accuracy or at all. This absence of data is especially pronounced for the stressstrain behavior and strength of the green body, both as a function of temperature and binder loading. In spite of these limitations, we use simple models that likely capture the essentials of the mechanical behavior of the green body during thermal debinding. For a linear heating rate, the pressure in the center of the green body increases at low temperature as binder is decomposed and then goes through a maximum before decreasing as binder is decomposed and more porosity is created. The stress in the green body then parallels the behavior in the pressure during the heating cycle. The strength of the green body, however, is a monotonically decreasing function with both decreases in the binder loading and increases in temperature. The intersection of the stress and strength profiles then determines if and

when in the heating cycle failure will occur. Several of the trends for when failure occurs are in qualitative agreement with what was observed in experiments for green body failure as a function of component size and heating rate.

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