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Lifetime prediction of structural ceramics by dynamic fatigue

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The dynamic fatigue behaviour of Al_2O_3 structural ceramics was studied by loading the stress on the specimen at a constant stress rate. The fracture strength of Al_2O_3 specimens under dynamic mode was constant up to a critical stress and then the lifetime abruptly decreased to zero as predicted theoretically for both the single cycle and the repeated loading. The material constant *A* was nearly constant and has no relation with loading mode and down speed for unnotched and notched specimen. The fracture strength obtained by theoretical calculation from the constants *n* and *A* was in good agreement with the measured value.

Key words: dynamic fatigue, stress rate, material constant *A*, *n*(crack growth exponent).

Introduction

The fracture of structural ceramics is generally known to be ruled by the slow crack growth mechanism. Many researchers have concentrated their efforts in materials science to study stress, strain, elastic modulus charac- teristics and also on fracture mechanics to study crack size, stress intensity factor, fracture probability, lifetime etc. Hence, these research fields have been much pro- gressed. However, the knowledge about bending strength, refractoriness, thermal expansion coefficient in classical and advanced ceramics are insufficient for design and application of structural ceramics (*e.g.* Al_2O_3, ZrO_2, SiC, Si_3N_4 etc.). For this reason, an investigation of the fatigue behaviour in structural ceramics becomes more important. Until now, however, most fracture characterizations have been achieved under static and cyclic load using glass ceramics which follows fracture theory very well [1, 2]. On the other hand, very few dynamic fatigue studies of structural ceramics have been made because too many specimens are needed for the test and the dynamic loading mode is not much applied to real usage. However, it is possible and efficient to examine various factors in short duration dynamic fatigue tests [3].

It was reported [4, 5] that the fracture strength of Al_2O_3 ceramics was constant up to a critical stress and then abruptly decreased in both static and cyclic fatigue tests. In the present study the dynamic fatigue behaviour of Al_2O_3 ceramics under various dynamic loading conditions as well as under repeated dynamic loading were observed using a four point flexural system in air at room temperature. The lifetime prediction and the precise measuring method of the material constants are discussed.

Static and cyclic fatigue lifetime of ceramics were predicted by the slow crack growth mechanism in the previous studies [4, 5] and it was confirmed that the predicted lifetime agreed well with the experimental data. In the present study, however, Al_2O_3 ceramic specimens were subjected to dynamic fatigue tests in which the Al_2O_3 specimens suffered under the four-point flexure system with various strain rates and various notch lengths to find the fatigue behaviour of Al_2O_3 ceramics and relate there to the fracture strength, the dynamic fatigue lifetime, the crack growth exponent and other constants.

Theoretical Background

The fatigue lifetime of ceramics is generally controlled by the slow crack growth mechanism [6], which can be expressed as Equation 1,

$$V=\frac{da}{dt}=AK_I^n \tag{1}$$

where a is the crack length, t the time, A the constant, K_I the stress intensity factor, and n the crack growth exponent. The stress intensity factor K_I is expressed as Equation 2,

$$K_I=Y\sigma\sqrt{a} \tag{2}$$

where σ is the applied stress and Y is the geometric factor. Fracture strength S can be derived from Equation 2 and can be written as Equation 3,

$$S=\frac{K_{IC}}{Y\sqrt{a}} \tag{3}$$

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Differentiation of Equation 3 with respect to time t gives Equation 4,

$$\frac{dS}{dt}=-\left(\frac{K_{IC}}{2Y}\right)a^{-3/2}\frac{da}{dt}=-\left(\frac{Y^2}{2K_{IC}^2}\right)S^3V \tag{4}$$

Substitution of Equations 3 and 4 into Equation 1 gives Equation 5,

$$V=AK_{IC}^n(\sigma/S)^n \tag{5}$$

Substitution of Equation 5 into Equation 4 results in Equation 6,

$$\frac{dS}{dt}=-(AY^2K_{IC}^{n-2}/2)(\sigma/S)^nS^3 \tag{6}$$

Integration of Equation 6 from time zero to t produces Equation 7,

$$S_t=\left[S_i^{n-2}-\frac{(n-2)AY^2K_{IC}^{n-2}}{2}\int_0^t[\sigma(t)]^n dt\right]^{1/(n-2)} \tag{7}$$

where S_i is initial fracture strength and S_t is the fracture strength when the load $\sigma(t)$ is applied to the specimen during the time t. In a previous study [7], the equivalent static stress σ_{es}, which is equal to the stress actually applied to the specimen during the time, was derived as shown in Equation 8,

$$\sigma_{es}=\left[t^{-1}\int_0^t[\sigma(t)]^n\ dt\right]^{1/n} \tag{8}$$

If the equivalent static stress of Equation 8 is applied to Equation 7, Equation 9 can be obtained,

$$S_t=\left[S_i^{n-2}-\frac{(n-2)AY^2K_{IC}^{n-2}}{2}\sigma_{es}^n\cdot t\right]^{1/(n-2)} \tag{9}$$

The values of S_i, K_{IC}, n and A were measured in this study. The value of the geometric factor Y was cited from the data reported by Sih [8]. It can be seen from Equation 9 that the strength of the specimen degrades as the loading time passes [9].

In the dynamic loading test, the equivalent static stress σ_{es} can be derived as Equation 10,

$$\sigma_{es}=\left(\frac{1}{n+1}\right)^{1/n}\dot{\sigma}\cdot t \tag{10}$$

where $\dot{\sigma}$ is the constant loading rate and can be expressed as $\dot{\sigma}=\sigma/t$. Substitution of Equation 10 into Equation 9 gives Equation 11. Equation 11 means that the strength of the specimen degrades by the dynamic loading $\dot{\sigma}$ and the time t.

$$S_t=\left[S_i^{n-2}-\frac{(n-2)AY^2K_{IC}^{n-2}\dot{\sigma}^n}{2(n+1)}t^{n+1}\right]^{1/(n-2)} \tag{11}$$

In the repeated dynamic loading test, the equivalent static stress σ_{es} can be derived as Equation 12,

$$\sigma_{es}=\left(\frac{1}{n+1}\right)^{1/n}\sigma_{max} \tag{12}$$

where σ_{max} is the maximum stress of one period of the constant loading rate for monotonically increasing stress in this study. Substitution of Equation 12 into Equation 9 gives Equation 13. Equation 13 implies that the strength of the specimen degrades by the maximum stress of the repeated dynamic loading and time t.

$$S_t=\left[S_i^{n-2}-\frac{(n-2)AY^2K_{IC}^{n-2}\sigma_{max}^n}{2(n+1)}t\right]^{1/(n-2)} \tag{13}$$

Experiments

Alumina was selected as the sample for this study because alumina is generally used for typical structural ceramic applications. Commercial Al_2O_3 powder (AES-11, Sumitomo, Japan) was isostatically pressed at 138 MPa and sintered at 1600°C for 1 h to prepare specimens 3 mm × 4 mm ×35 mm through cutting and grinding.

For the single cycle dynamic loading test, the four-point bending strength was measured for unnotched and 0.5 mm notched specimens at the 0.001 and 0.0005 mm/min down speed, respectively, after loading the stress by a constant stress rate from 0% up to over the range of 0% to 105% of the average inert strength, as shown in Fig. 1 for the first cycle. However, for the repeated dynamic loading test, the four-point bending strength was measured for 0.5 mm notched specimens at the 0.001 and 0.0005 mm/min down speed, respectively, after 95% of the average inert strength was repeatedly applied as shown in Fig. 1. The stress rates were 0.0646 and 0.03045 $MPa\cdot s^{-1}$, respectively, for the down speeds 0.001 and 0.0005 mm/min.

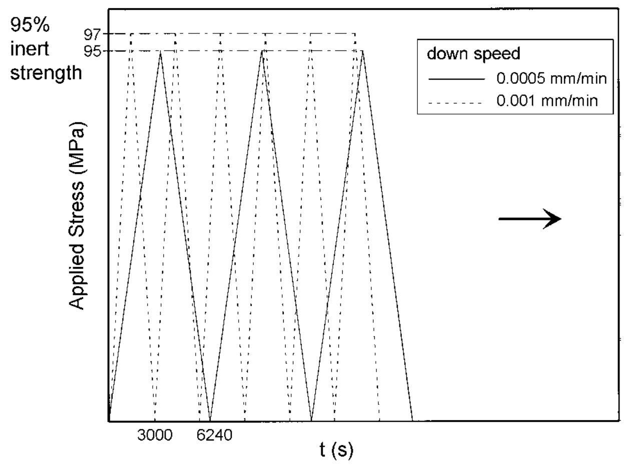

Fig. 1. Schematic diagram of repeated dynamic loading.

Table 1. Properties of Alumina Specimens

Property	Mean Value	Standard Deviation
Density	3.91 g/cm^3	0.01
4 Point Bending Strength	360 Mpa	23
Fracture Toughness	3.91 Mpam$^{1/2}$	0.15
Young's Modulus	330 Gpa	29

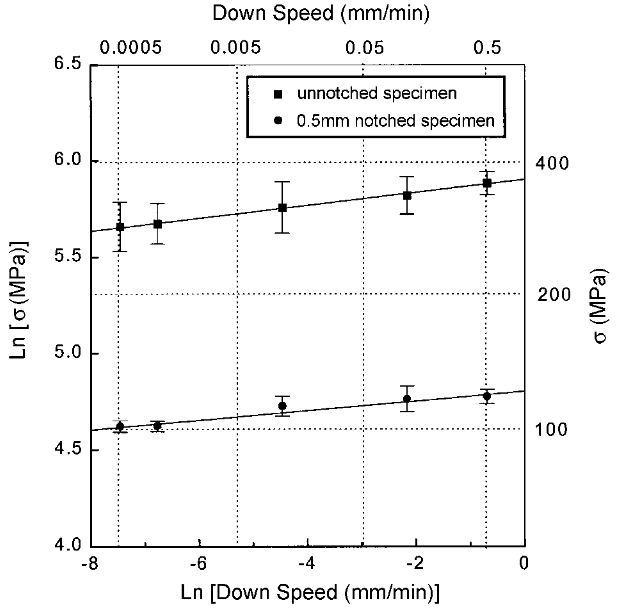

Fig. 2. Fracture strength of alumina ceramics as a function of down speed.

Results and Discussion

Density, four-point flexural strength, fracture toughness, Young's modulus, and Weibull modulus of the Al_2O_3 specimen were measured and are given in Table 1.

Figure 2 shows the measured fracture strength of unnotched and 0.5 mm notched specimens as a function of down speed. The unnotched specimens show a higher strength than the 0.5 mm notched specimens and the specimens which were fatigued at the slower down speed show a lower fracture strength because the specimen fatigued at the slower down speed suffered fatigue for a longer time than the specimen fatigued at the faster down speed. It can be seen from Fig. 2 that the fracture strength data of 0.5 mm notched specimen shows less deviation than those of the unnotched specimen.

Figure 3 and Fig. 4 show the residual fracture strength of the unnotched specimens which were measured after loading at a constant stress rate from 0% up to 100% of the average inert strength at the down speed of 0.001 ($\dot{\sigma}$=0.0646 MPa/s) and 0.0005 mm/min ($\dot{\sigma}$=0.03045 MPa/s) respectively. The unnotched specimens may have many vague flaws which

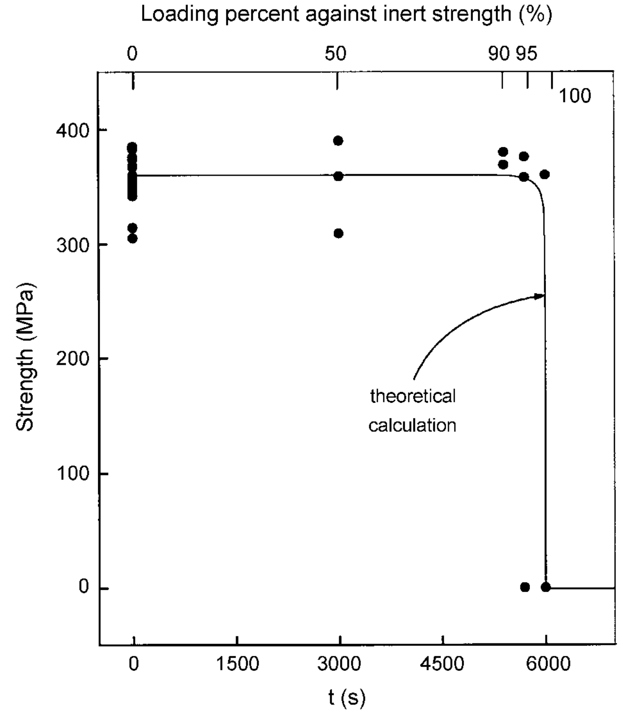

Fig. 3. Fracture strength of the unnotched alumina ceramics as a function of time under the single cycle dynamic loading with 0~100% inert strength at 0.001 mm/min down speed.

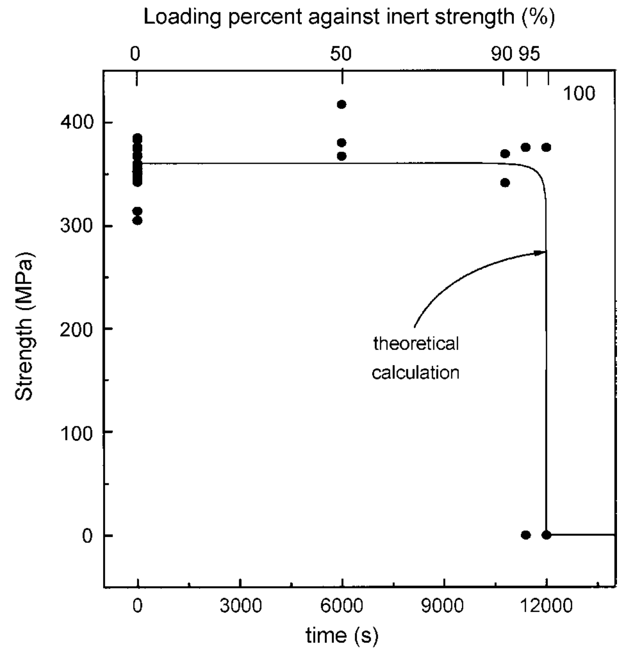

Fig. 4. Fracture strength of the unnotched alumina ceramics as a function of time under the single cycle dynamic loading with 0~100% inert strength at 0.0005 mm/min down speed.

were introduced during the specimen fabrication process. Hence, some specimens were fractured under the load of 95% inert strength and some others were fractured under the load of 105% inert strength. Therefore, it was very difficult to control the loading of unnotched specimens. For this reason a notch of 0.5 mm was introduced into the specimen to control the loading more precisely. Solid curves in Fig. 3 and Fig. 4 represent the theoretical calculations using Equation 11. From these curves, it can be appreciated that the fracture strength of Al_2O_3 specimens loaded under dynamic mode is maintained to the critical applied stress and then the lifetime abruptly decreases to zero. The measured values are in good agreement with the theoretically-calculated lines, as shown in Figs. 3 and 4. From Figs. 3 and 4 it can be seen that the lifetime for the cross-head speed 0.0005 mm/min is about twice the value for the cross-head speed 0.001 mm/min, because the stress rate of the latter is about twice that of the former.

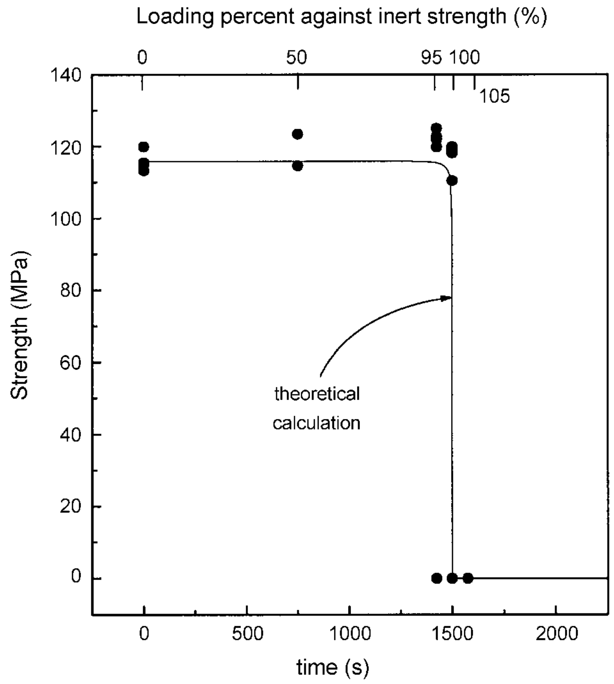

Fig. 5. Fracture strength of the 0.5 mm notched alumina ceramics as a function of time under the single cycle dynamic loading with 0~105% inert strength at 0.001 mm/min down speed.

Figure 5 and Fig. 6 show the residual fracture strength of 0.5 mm notched specimens, which were measured after loading the stress by a constant stress rate from 0% up to 105% of the average inert strength, at the down speeds of 0.001 and 0.0005 mm/min, respectively. The notch was assumed to be an initial crack in the specimen. The deviation of fracture strengths of the notched specimens was, therefore, smaller than the unnotched ones. A few specimens were fractured under the load of 95% inert strength. All specimens were fractured under the load of 105% inert strength. Solid curves in Figs. 5 and 6 were obtained by theoretical calculation using Equation 11. The fracture strength of the notched Al_2O_3 specimen loaded under the dynamic mode was almost constant up to the critical applied stress and then the lifetime abruptly decreased as in the case of the unnotched specimens. The measured values are in good agreement with the theoretically-obtained curves. From Figs. 5 and 6 it can be understood that the lifetime for the down speed 0.0005 mm/min is about twice the value for the down speed 0.001 mm/min, because the stress rate of the latter is about twice that of the former.

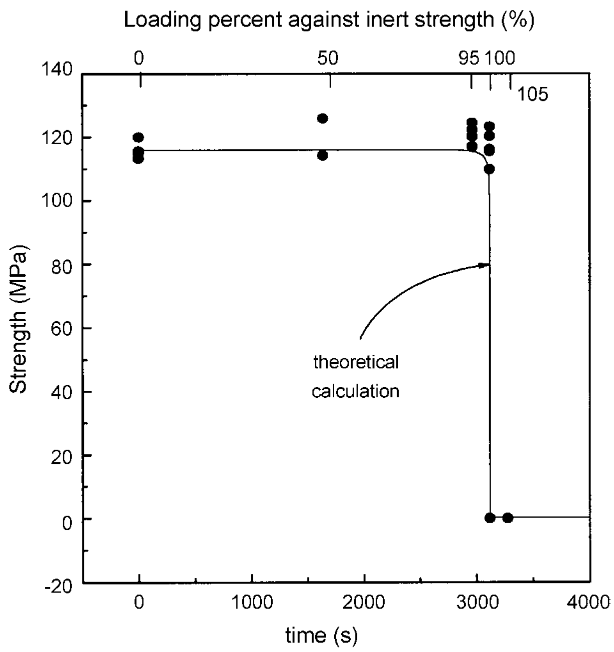

Fig. 6. Fracture strength of the 0.5 mm notched alumina ceramics as a function of time under the single cycle dynamic loading with 0~105% inert strength ar 0.0005 mm/min down speed.

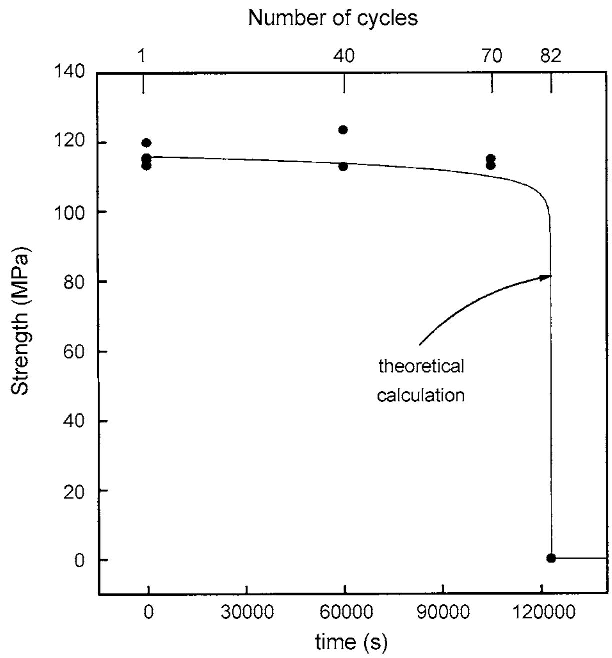

Fig. 7. Fracture strength of the 0.5 mm notched alumina ceramics as a function of time under the repeated dynamic loading with 95% inert strength ar 0.001 mm/min down speed.

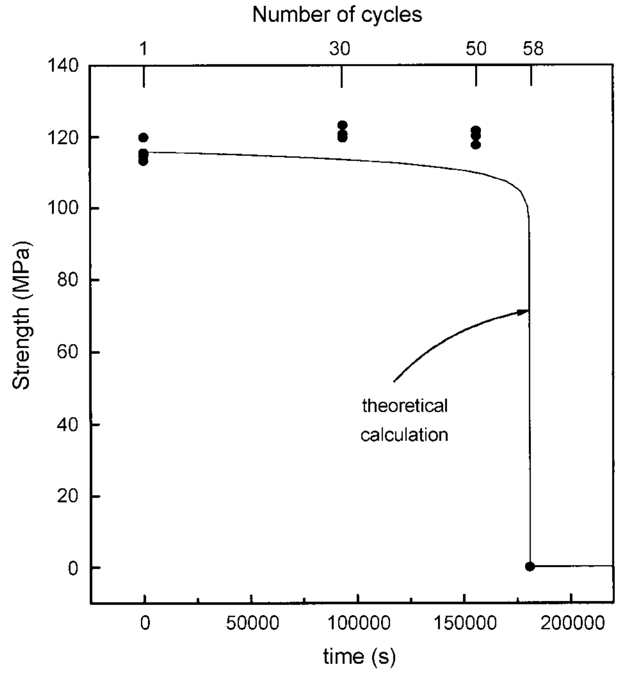

Fig. 8. Fracture strength of the 0.5 mm notched alumina ceramics as a function of time under the repeated dynamic loading with 95% inert strength ar 0.0005 mm/min down speed.

Figures 7 and 8 show the residual fracture strength of 0.5 mm notched specimens, which were measured after repeated loading at a constant stress rate from 0% to 95% of the average inert strength at the down speeds of 0.001 and 0.0005 mm/min, respectively. The specimens were fractured after 82 cycles for the down speed of 0.001 mm/min, on the other hand, the specimens were fractured after 58 cycles for the down speed of 0.0005 mm/min. Hence, the residual fracture strength was measured after loading 40 and 70 cycles for 0.001 mm/min down speed, and measured after loading 30 and 50 cycles for 0.0005 mm/min down speed. Solid curves in Figs. 7 and 8 were obtained by theoretical calculation using Equation 13. The fracture strength obtained by calculation from the constants *n* and *A* using Equation 13 was in good agreement with the measured values as shown in Figs. 7 and 8. From Figs. 7 and 8 it can be seen that the lifetime for the down speed 0.0005 mm/min is about twice the value for the down speed 0.001 mm/min, because the stress rate of the latter is about twice that of the former.

Material constant *A* was calculated from these data using Equations 11 and 13, and the crack growth exponent *n* was calculated from the data given in Fig. 2. Values for *A* and *n* are given in Table 2.

Generally, material constant *A* has the same value when the materials have the same initial crack length and same shape. For the specimens loaded at 0.001 mm/min down speed, the values of constant *A* were 1.73×10^{-202} and 2.15×10^{-259} for the unnotched and 0.5 mm notched ones, respectively. For the specimens

Table 2. The Variation of Constant *n* and *A* according to Notch Length

Notch Length (mm)		*n*	*A*	
			Down Speed	Down Speed
0	Single Cycle	29.81	1.73×10^{-202}	1.25×10^{-202}
0.5	Single Cycle	38.76	2.15×10^{-259}	5.38×10^{-259}
	Repeated		1.92×10^{-260}	6.78×10^{-260}

loaded at 0.0005 mm/min down speed, they were 1.25×10^{-202} and 5.38×10^{-259} for the unnotched and 0.5mm notched ones, respectively. For the specimens repeatedly loaded at 0.001 mm/min down speed, the value of constant *A* was 2.00×10^{-260} and for the specimens repeatedly loaded at 0.0005 mm/min down speed, it was 5.88×10^{-260}. The values of *A* are nearly the same and have no relation with loading mode and down speed for unnotched and notched specimens. From this result it can be expected that the theoretical equation introduced in this study may give the right information for the real residual fracture strength.

Conclusions

(1) The fracture strength of Al_2O_3 specimen under the dynamic mode was constant up to the critical applied stress and then the lifetime abruptly decreased to zero for both the single cycle and the repeated loading.

(2) The material constant *A* was nearly constant and had no relation with loading mode and down speed for unnotched and notched specimens.

(3) The fracture strength obtained by theoretical calculation from the constants *n* and *A* was in good agreement with the measured value.

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