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# A temperature control system for ceramic drying furnaces

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An electric furnace, inside which desired temperatures are kept constant by generating heat, is known to be a difficult system to control and model exactly because system parameters and response delay time are varied as the temperature and position are changed. In this study, the heating system of ceramic drying furnaces with time-varying parameters is mathematically modeled and control parameters are estimated by using a recursive least-square method. The generalized predictive control with exponential weight (GPCEW), which always guarantees the stability of closed loop systems and can be effectively applied to internally unstable systems, is employed in the temperature control of ceramic drying electric furnaces and its performances is experimentally verified. It is proven that temperature tracking of GPCEW is more stable than the generalized predictive control (GPC) and rapidly settles down by increasing the prediction horizon.

Key words: Generalized Predictive Control with Exponential Weight, Ceramic Drying Electric Furnace, Adaptive Predictive Control, Recursive Least Squares Method.

#### Introduction

Ceramic products are usually dried under conditions where temperature and moisture are kept constant during the drying process. When the drying speed is too fast or not steady, ceramic products crack due to the residual stress caused by the un-even shrinkage and the high pressure. In order to avoid this mechanical defect; therefore, a temperature control based on a knowledge of the drying temperature distribution inside the furnace may be needed.

An electric furnace used for drying green ceramics is a system that controls the furnace interior to a fixed temperature with the heat supplied by a heat generation unit. Since the electric furnace system has the characteristics where system parameters and response delay time vary as the surrounding and control temperatures change, it is difficult to model and control the system accurately.

Although a PID (Proportional, Integral, and Derivative) control is generally used in the processes controlling the temperatures and most industrial fields, it has a disadvantage because workers have to specify and tune up the PID control parameters, based on their experience, whenever the dynamic characteristics of the process or environmental conditions change. However, an adaptive control automatically finds the self-tuning parameters associated with the changes in the dynamic characteristics of the process and environmental conditions.

Since the 1970s, the long range predictive control commonly used to model prediction control has employed the receding horizon control method [1-6]. On the other hand, generalized predictive control (GPC) [7, 8] is generally known to be the most effective control method in for those predictive control fields. GPC has the advantage that it can work well even in the conditions where real processes are difficult to control.

Although GPC has the advantage of practical availability, its stability analysis is performed using unlimited area samples. GPC cannot be employed in systems that have samples in a limited area only and which are unstable. In order to overcome this sampling problem so that the GPC can always guarantee a control function even in a limited area, many investigationhave been carried out [9-12]. The generalized predictive control with exponential weight (GPCEW) maintains the stability of a closed loop system and effectively works in a internally unstable system.

In this study, the control law called by a generalized predictive control with exponential weight (GPCEW) is applied to the temperature control system of a ceramic drying electric furnace and has been experimentally verified by showing temperature tracking performance.

# Generalized predictive control with exponential weight

Predictive control of a model composes predictive equations based on the model used. It is very important to determine the model effectively, because it affects

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the robustness of the system according to the accuracy of the description of the model. CARIMA (controlled auto-regressive integrated moving average) model is considered in GPC as follows:

$$A(q^{-1})y(t) = B(q^{-1})u(t-1) + C(q^{-1})\xi(t)/\Delta$$
(1)

where are y(t) is the measured output, u(t) is the control input,  $A(q^{-1}), B(q^{-1})$  and  $C(q^{-1})$  are sampling intervals which are defined as  $t = 0, 1, 2, \dots$ . The polynomials are expressed by backward shift operators as follows :

$$A(q^{-1}) = a + a_1 q^{-1} + a_2 q^{-2} + \dots + a_n q^{-n}$$
(2)

$$B(q^{-1}) = b_0 + b_1 q^{-1} + b_2 q^{-2} + \dots + a_m q^{-m}$$
(3)

$$C(q^{-1}) = 1 + c_1 q^{-1} + c_2 q^{-2} + \dots + c_n q^{-n}$$
(4)

and  $\Delta = 1 - q^{-1}$ . Furthermore,  $\xi(t)$  is an uncorrelated random sequence.

A diophantine equation for a generalized CARIMA model is written as follows :

$$C(q^{-1}) = E_j(q^{-1})A\Delta + q^{-1}F_j(q^{-1})$$
(5)

In order to make the model simple,  $C(q^{-1}) = 1$  is assumed. Then Eq. (5) is written as follows :

$$1 = E_j(q^{-1})A\Delta + q^{-1}F_j(q^{-1})$$
(6)

where  $E_j(q^{-1})$  and  $F_j(q^{-1})$  are the polynomials uniquely determined by  $A(q^{-1})$  and the degree of  $E_j(q^{-1})$  is j-1.  $E_j$  and  $F_j$  are obtained by the equation next to the recursive Diophantine equation Eq. (6) :

$$1 = R_j(q^{-1})A\Delta + q^{-1}S_j(q^{-1})$$
(7)

where  $R = E_{j+1}$ ,  $S = F_{j+1}$  (j = 1, 2, ..., N).

Subtracting Eq. (6) from Eq. (7), Eq. (8) is obtained :

$$0 = (R_j - E_j)A\Delta + q^{-j}(q^{-1}S_j - F_j)$$
(8)

Eq. (8) can be rewritten as follows :

$$\tilde{A}\tilde{R} + q^{-j}(q^{-1}S - F + \tilde{A}r_j) = 0$$
(9)

where  $\tilde{A} = A\Delta$ . In Eq. (9), as  $q^{-j}(q^{-1}S - F + \tilde{A}r_j)$ included the terms of  $q^{-j}$ ,  $q^{-(j+1)}$ , ...., the (j-1)th term of  $\tilde{AR}$  is zero,  $\tilde{R} = 0$ , and  $q^{-j}(q^{-1}S - F + \tilde{A}r_j) = 0$ . Solving S in Eq. (9),

$$S = qf_0 + f_1q^{-1} + \dots + r_j + \tilde{a}_1r_jq^{-1} + \tilde{a}_{na+1}r_jq^{-(na+1)}$$
(10)

Hence, from the coefficient of the q term in Eq. (10),  $f_0 = r_i$ ,  $S_i = f_{i+1} - a_{i+1}r_i$  ( $i = 0 \sim \text{deg}S$ ) are obtained. The control law minimizes the cost function defined by the following form :

$$J = \left\{ \sum_{j=0}^{N_2} \mu(j) [y(t+j) - w(t+j)]^2 + \sum_{j=0}^{N_2} \lambda(j) [\Delta u(t-j+1)]^2 \right\}$$
(11)

where y(t+j) is a future plant output, w(t+j) is a future setting point,  $N_1$  is a minimum prediction horizon,  $N_2$  is the maximum prediction horizon,  $\mu(j)$   $\rho(j)$  are control weights. The choice  $N_u = 1$  of makes the calculation process tremendously simple. Rewriting Eq. (11) in vector form,

$$J = E \{J(N_1, N_2) \\ = E \{(Y - W)^T (Y - W) + \lambda U^T U\}$$
(12)

is obtained.

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For given exponential weights to the tracking error and control input, the following forms can be considered :

$$\mu(i) = \tilde{\eta} \dot{\beta} \quad \text{for tracking error} \\ \rho(i) = \tilde{\rho} \dot{\beta} \quad \text{for control increments}$$
(13)

If  $\mu=1$  and  $\alpha = \beta^{-1/2}$  are defined to improve the efficiency of the control, Eq. (13) can be rewritten as follows :

$$\mu(i) = \tilde{\eta}\beta^{-2i}, \,\rho(i) = \tilde{\rho}\beta^{-2i} \tag{14}$$

In Eq. (12)

$$Y = G\Delta U + F \tag{15}$$

$$\Delta U = (G^T M G + \Lambda I)^{-1} G^T M (W - F)$$
(16)

where

$$Y = [y(t+N_1) \ y(t+N_1+1) \ \dots \ y(t+N_2)]^T$$
(17)

$$\Delta U = \left[\Delta u(t) \ \Delta u(t+1) \ \dots \ \Delta u(t+N_2-1)\right]^T$$
(18)

$$F = [f(t+N_1) f(t+N_1+1) \dots f(t+N_2)]^T$$
(19)

$$M = diag[\alpha^{-1}, \dots, \alpha^{-2i}, \alpha^{-2N_u}]$$
(20)

$$\Lambda = \rho \cdot diag[1, \, \alpha^{-1}, \, \dots, \, \alpha^{-2i}, \, \alpha^{-2(N_u - 1)}]$$
(21)

In Eq. (15),  $G\Delta U$  and F are the forced response of the process and the free response of the process, respectively. The forced response is generated by the control input which has not been decided. The free response is calculated by the past u already known and y of the present and past times. G is a lower-triangular

$$G = \begin{bmatrix} g_{N_1-1} \dots g_0 \dots & 0 \\ g_{N_1} & \dots & g_0 \dots & 0 \\ \vdots & \dots & \vdots \\ g_{N_1-1} & \dots & g_{N_2-N} \end{bmatrix}$$
(22)  
where  $g_j = -\sum_{i=0}^j a_i g_{j-1} + \sum_{i=0}^{j-1} b_i$  and  $j = 1, 2, \dots, N_2$ .

## Mathematical modeling of the electric furnace

#### Modeling

The modeling of the thermal process of an electric furnace heating system can be developed from the basic mechanisms of heat transfer, namely conduction, convection, and radiation. However, to simplicity the modeling and the usage in the control algorithm, the modeling includes only convection. The energy balance equation can be derived as follows:

$$C\frac{dy(t)}{dt} = p(t) - \{y(t) - y_c(t)$$
(23)

where C is the thermal capacitance, R is thermal resistance, p(t) is the power supplied, y(t) is the inlet and temperature,  $y_c(t)$  is the ambient temperature. Eq. (23) can be transformed into Eq. (24) by a Laplace transformation as follows :

$$Y(s) = \frac{1}{C\left(1 + \frac{1}{RC}\right)} \left(P(s) + \frac{Y_c(s)}{R}\right)$$
(24)

where

$$U(s) = P(s) + \frac{Y_c(s)}{R}$$
(25)

Therefore, the transfer function G(s) is derived as follows :

$$G(s) = \frac{Y(s)}{U(s)} = \frac{R}{RCs+1} = \frac{R}{Ts+1}$$
(26)

where U(s) is the amount of heat input, output Y(s) is the temperature output, and time constant T is the multiple of R and C.

Hence, if ZOH (zero order hold) is used, the discrete the transfer function  $G(z^{-1})$  is obtained by:

$$G(z^{-1}) = (1 - z^{-1})Z \left[ \frac{1}{C\left(s + \frac{1}{RC}\right)} \right]$$
(27)

$$= (1-z^{-1}) \left[ R \left( \frac{1}{1-z^{-1}} - \frac{1}{1-e^{-T/RC} z^{-1}} \right) \right] = \frac{bz^{-1}}{1+az^{-1}}$$

where  $b = R(1-e^{-T/RC})$  and  $a = e^{-T/RC}$ .

#### Parameter Estimation

In order to design dynamic control systems, it is very important to determine a model, which can describe the dynamic characteristics. A system identification is used to determine the model using experimental data. During the system identification process, 3 points are considered. Firstly, it should have the least number of parameters. Secondly, the parameters should be uniquely determined by the observation. Finally, the control design should be easy and simple.

Generally, the estimation function for a parameter identification J is expressed as follows:

$$J = \sum_{k=1}^{N} f(e(k, \theta))$$
(28)

In this paper, an electric furnace heating system is assumed as a ARMAX (Auto Regressive Moving Average with exogenous inputs) model and system parameters are estimated by RLS (Recursive leastsquare). Furthermore, in controller design, the control input is determined using estimated parameters. The ARMAX model for the electric furnace heating system is written as follows:

$$A(q^{-1})y(t) = B(q^{-1})u(t) + c(q^{-1})e(t)$$
(29)

where e(t) is the estimation error and

$$A(q^{-1})y(t) = 1 + a_1q^{-1} + \dots + a_nq^{-n}$$
(30)

$$B(q^{-1})y(t) = b_0 q^{-1} + b_1 q^{-2} + \dots + b_m q^{-m}$$
(31)

If  $C(q^{-1})$  is assumed to be 0 (zero) in Eq. (29), Eq. (29) can be rewritten as follows:

$$y(t+1) = -a_1y(t) - a_2(t-1) - \dots - a_ny(t-n+1) + b_0y(t) + b_1u(t+1) + \dots + b_mu(t-m)$$
(32)

Supposing the discrete transfer function of the plant to be a first order system and expressing Eq. (32) in matrix form, Eq. (33) is obtained:

$$y(t+1) = [-a \ b] \begin{bmatrix} y(t-1) \\ u(t-1) \end{bmatrix} = \theta^T \phi(t-1)$$
(33)

where the parameter vector  $\hat{\theta}$  and measurement vector  $\hat{\phi}$  vector are expressed as follows:

$$\hat{\theta}^{T} = [a_1, a_2, \dots, b_1, b_2, \dots]$$
(34)

$$\hat{\phi}^{T} = [u(t-1), u(t-2), \dots, y(t-1), y(t-2), \dots]$$
(35)

#### **Experiments**

As shown in Fig. 1, an electric furnace temperature control system consists of five modules: an electric furnace unit containing heating elements, a temperature sensing module employing a K-type thermocouple, a board for A/D conversion and digital output, a solid state relay (SSR) for controlling heat power, and a personal computer adjusting the SSR output and showing graphically the performance.

A firebrick,  $9.5 \text{ cm} \times 11.5 \text{ cm} \times 6.5 \text{ cm}$ , is heated. Six thermocouples are located at the center of each side of the brick. The thermocouples can be changed in length so that it is possible to measure temperatures in various positions.

Generally, temperature control needs a long time because the response of a temperature control system is delayed. In this system, the cooling process is operated by natural convection.

The analysis data was gathered every 10 seconds and 100 measured temperatures were averaged to reduce errors in sensing. As the coefficients of input and output parameters were not known, one of the parameters and one of the b parameters had be estimated. The effect of the design parameters  $N_u$ ,  $N_1$  and  $N_2$  were known from the experiment. In this experiment,  $\lambda = 1$ , P(0) = 1000I and  $\hat{\theta}(0) = 0$  were initially defined.

Figure 2 shows the response of the GPC when  $N_1=1$ ,  $N_2=1$ ,  $N_u=1$ , and  $\lambda=1$ . When the initial temperature was 28 °C and the setting point, 150 °C, was kept until the response was measured at 5200 seconds, the initial overshoot, rise time, steady state error, and settling time were 11.4 °C, 930 seconds, 0.81 °C, 3430 seconds, respectively. When the setting point was changed to 300 °C at 5200 seconds and kept until the measurements were finished, the overshoot, rise time, settling time, steady state error were 19.29 °C, 990 seconds, 2740 seconds, and 0.91 °C, respectively.

Figure 3 shows the response of GPCEW in the special case shown in Fig. 2. The weighting parameters were set as  $\alpha = 1$ ,  $\rho = 0$ . When the setting point was kept at 150 °C until the measuring time was 5200



Fig. 1. Schematic diagram of a furnace control system.



**Fig. 2.** GPC (generalized predictive control). Tuning parameter:  $N_1=1$ ,  $N_2=1$ ,  $N_u=1$ ,  $\lambda=1$ . Initial temp: 28.16 °C, Rise time: 780s, Overshoot: 13 °C



**Fig. 3.** GPCEW (generalized predictive control with exponential weighting). Tuning parameter:  $N_1=1$ ,  $N_2=1$ ,  $N_u=1$ ,  $\lambda=1$ . Weighting parameter:  $\alpha=1$ ,  $\rho=0$ . Initial Temp: 27.6 °C, Rise Time: 840 s. Overshoot: 2.9 °C

seconds, the initial overshoot, rise time, settling time, steady-state error were  $10.14 \,^{\circ}$ C, 940 seconds, 2920 seconds, and  $0.8 \,^{\circ}$ C, respectively. When the setting point was varied to 300  $\,^{\circ}$ C at 5200 seconds, the overshoot, rise time, settling time, and steady-state error were  $18.55 \,^{\circ}$ C, 1070 seconds, 3350 seconds and  $0.8 \,^{\circ}$ C, respectively.

When the design parameters were changed to  $N_1$ =1,  $N_2$ =10,  $N_u$ =2, the responses of GPC and GPCEW are shown in Fig. 4 and Fig. 5, respectively. When the setting point was 150 °C, the initial overshoot, rise time, settling time, and steady state error in GPC were 5.57 °C, 1570 seconds, 3030 seconds, and 0.68 °C, respectively. In GPCEW, however, they were 5.85 °C, 1140 seconds, 2700 seconds, and 0.48 °C, respectively. When the setting point was changed to 300 °C at the measuring time 5200 seconds, the overshoot, rise time, settling time, steady state error in GPC were 15.38 °C, 1000 seconds, 2350 seconds, 0.9 °C, respectively. However, the overshoot, rise time, settling time, steady state error in GPC were 6.61 °C, 1140 second, 1960 seconds, 0.67 °C, respectively.



**Fig. 4.** GPC (generalized predictive control). Tuning parameter:  $N_1=1, N_2=10, N_u=2, \lambda=1$ . Initial temp: 36.19 °C, Rise time: 800 s. Overshoot: 4.68 °C



**Fig. 5.** GPCEW (generalized predictive control with exponential weighting). Tuning parameter:  $N_1=1$ ,  $N_2=10$ ,  $N_u=2$ ,  $\lambda=1$ . Weighting parameter:  $\alpha=1$ ,  $\rho=0$ . Initial temp: 29.94 °C, Rise time: 960 s. Overshoot: 4.2 °C

As seen in the results above, when the control horizon  $N_u$  was increased in GPC, the overshoot and settling time were decreased by 5.83 °C, 220 seconds, respectively while the rise time was increased by 640 seconds. When the  $N_u$  was increased in GPCEW, however, the overshoot, rise time, and settling time were decreased by 1.3 °C, 120 seconds, 100 seconds, respectively.

Comparing the responses between GPC and GPCEW when the control interval  $N_u$  was increased and the weight parameter  $\alpha$  was given, although the rise time of GPCEW became a little bigger, the overshoot and settling time were greatly decreased. The GPCEW in considering a time delay made the rise time less that when the time dely was neglected.

### Conclusions

An electric furnace heating system, which operates slowly and has time-varying control parameters, was mathematically modeled. Then, a GPCEW control algorithm was employed to control the drying temperature of a ceramic electric furnace. The control parameters were estimated by a recursive least-square method. From this research, the following conclusions were obtained:

Although the rise time of GPCEW was increased a little more than that of GPC, the overshoot, settling time, and steady state error were decreased so that it can effectively control furnace temperatures.

While the increase of the prediction horizon in GPCEW makes the rise time a little bigger, the overshoot, steady state error, and settling time became smaller.

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